Gaussianity test of Planck CMB polarization data

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Why do we need a Gaussianity test?

- Inflation predicts Gaussianity of initial perturbations

- General Gaussianity of cosmological E and B modes (in the first approximation)

- Initial non-Gaussianity (refinement of the inflation model)

$$\phi = \phi_g + f_{NL}\phi_g^2$$

- Presence of unremoved non-Gaussian foregrounds (data cleaning)

Tests for Gaussianity:

- Minkowski functionals
- High-order correlations
- Kurtosis and skewness
- Statistics of local extrema
- Clusterization of peaks
- Percolation

How to provide Gaussianity test for linear polarization?

Stokes parameters:

$$Q = \left[\frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2}\right] E - 2\frac{\partial^2}{\partial x \partial y} B,$$

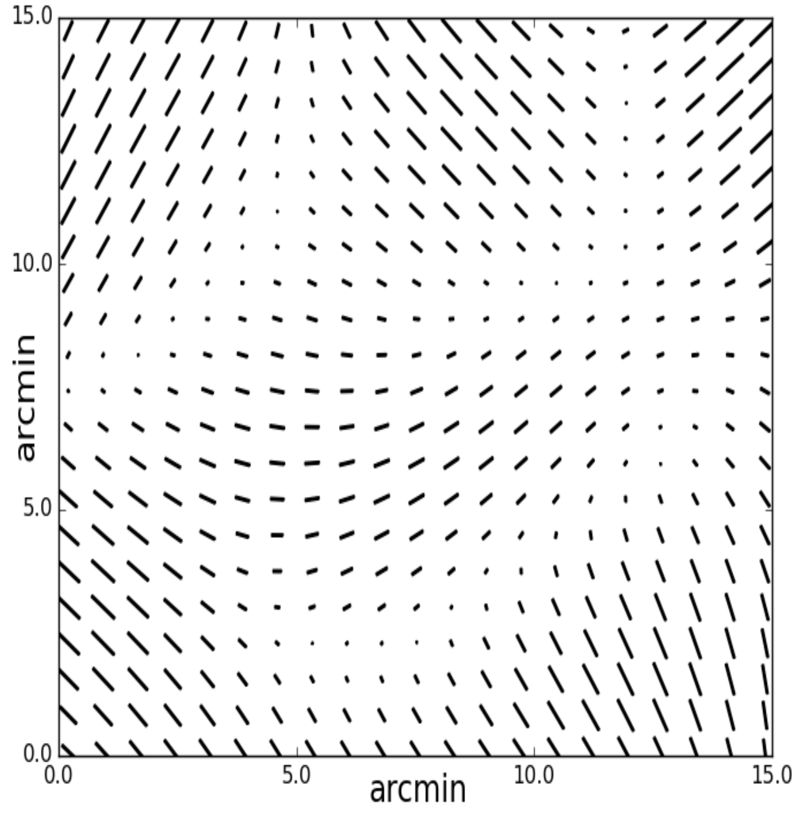
$$U = 2\frac{\partial^2}{\partial x \partial y} E + \left[\frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2}\right] B$$

$$\frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2} = \frac{\partial^2}{\partial \theta^2} - \frac{\cos\theta}{\sin\theta}\frac{\partial}{\partial\theta} - \frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\varphi^2},$$

$$\frac{\partial^2}{\partial x \partial y} = \frac{1}{\sin^2\theta}\frac{\partial}{\partial\varphi}\left[\sin\theta\frac{\partial}{\partial\theta} - \cos\theta\right],$$

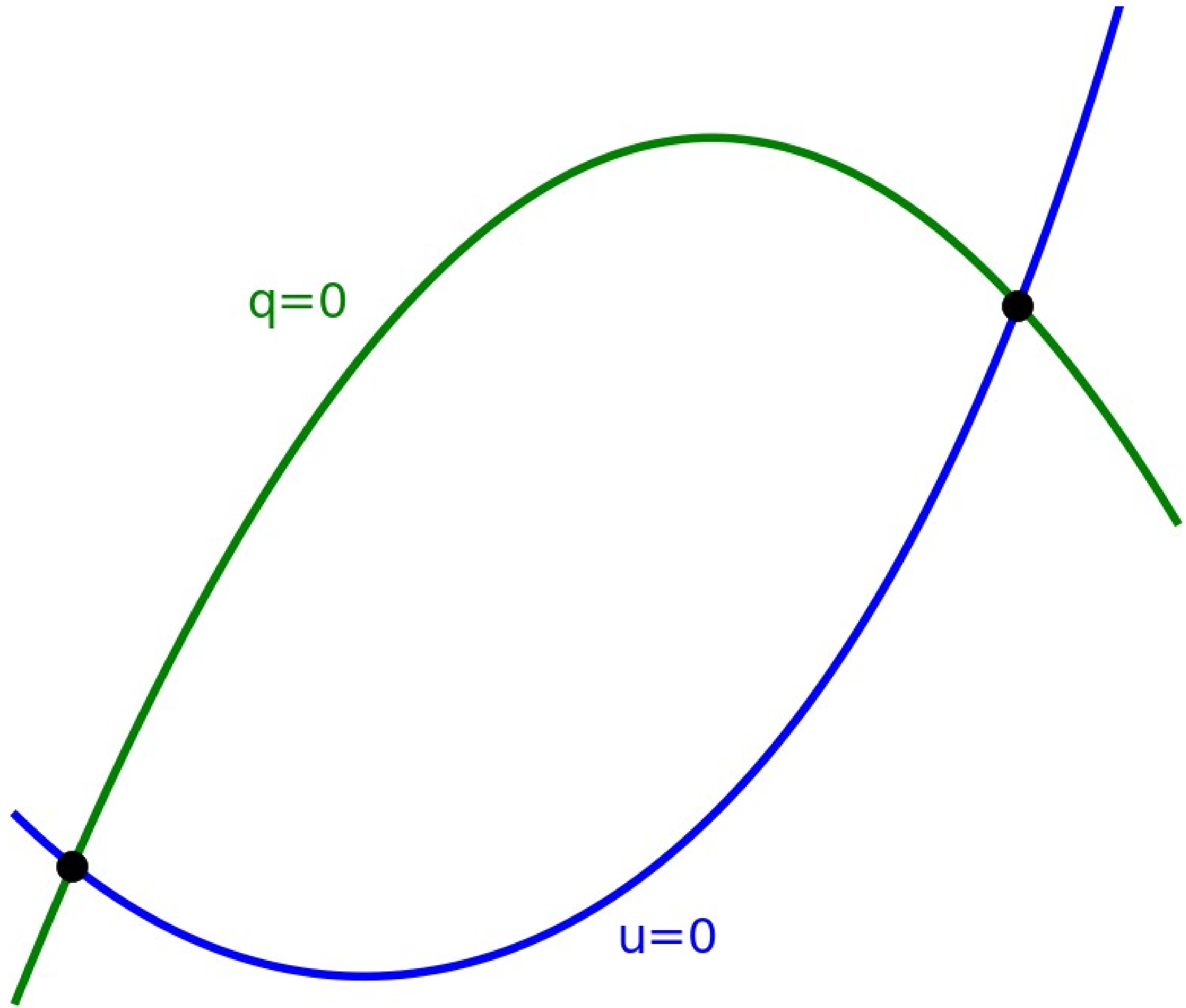
$$E, B = \sum_{\ell=2}^{\ell_{max}} \sum_{m=-\ell}^{\ell} a_{\ell m}^{E,B} \cdot Y_{\ell m}(\theta, \varphi)$$

Polarization picture



$$P^2 = Q^2 + U^2, \qquad \tan(2\phi) = U/Q.$$

Unpolarized points:



Spectral parameters and dimensionless values:

$$\langle P^2 \rangle = \langle Q^2 \rangle + \langle U^2 \rangle = 2\sigma_0^2,$$

$$\sigma_0^2 = \sum_{l=2}^{\ell_{max}} (2\ell+1)\frac{(\ell+2)!}{(\ell-2)!}(C_\ell^E + C_\ell^B),$$

$$\langle Q_x^2 \rangle + \langle Q_y^2 \rangle = \langle U_x^2 \rangle + \langle U_y^2 \rangle = \sigma_1^2,$$

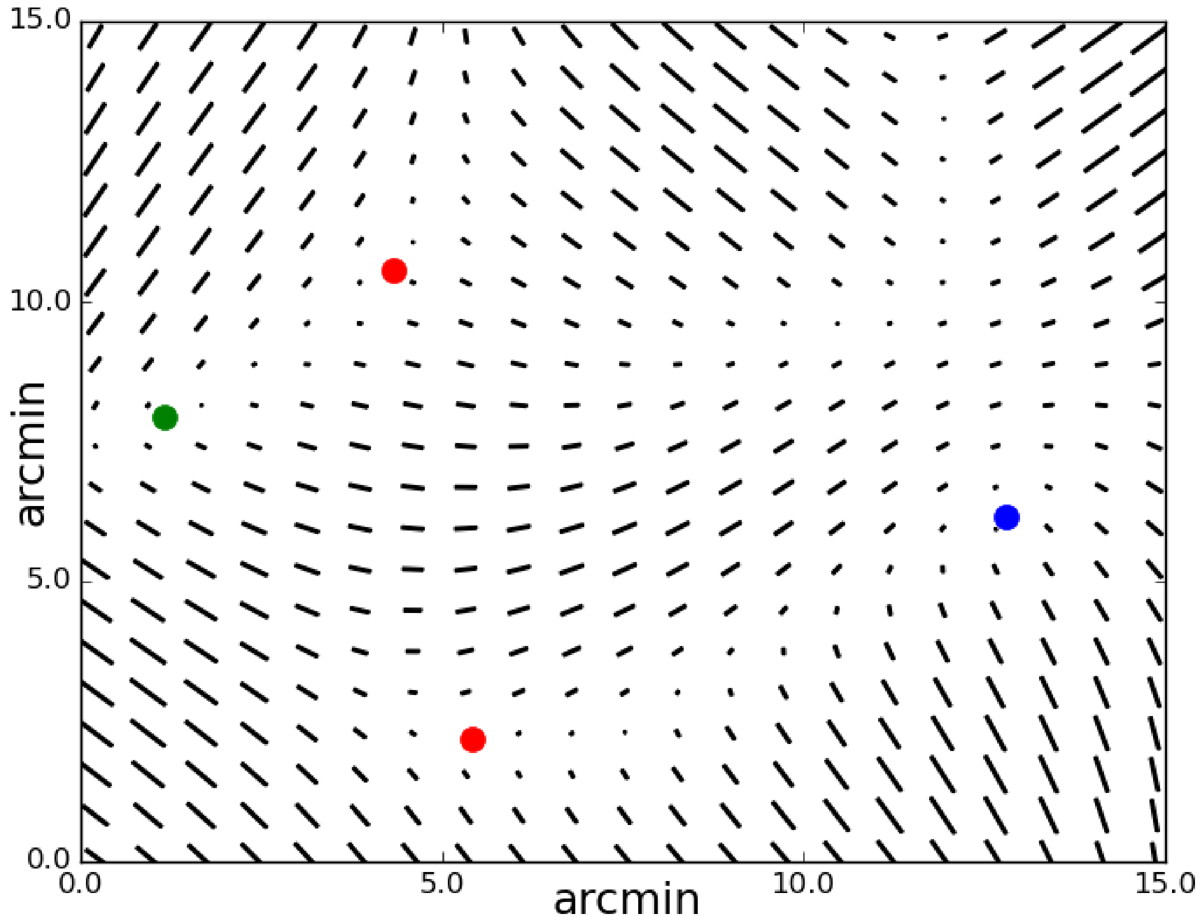
$$\sigma_1^2 = \sum_{l=2}^{\ell_{max}} (2\ell+1)\frac{(\ell+3)!}{(\ell-3)!}\left(C_\ell^E + C_\ell^B\right)$$

$$q = \frac{Q}{\sigma_0}, \quad u = \frac{U}{\sigma_0}, \quad p = \frac{P}{\sigma_0}$$

$$q_x = \frac{Q_x}{\sigma_1}, \quad q_y = \frac{Q_y}{\sigma_1}, \quad u_x = \frac{U_x}{\sigma_1}, \quad u_y = \frac{U_y}{\sigma_1}$$

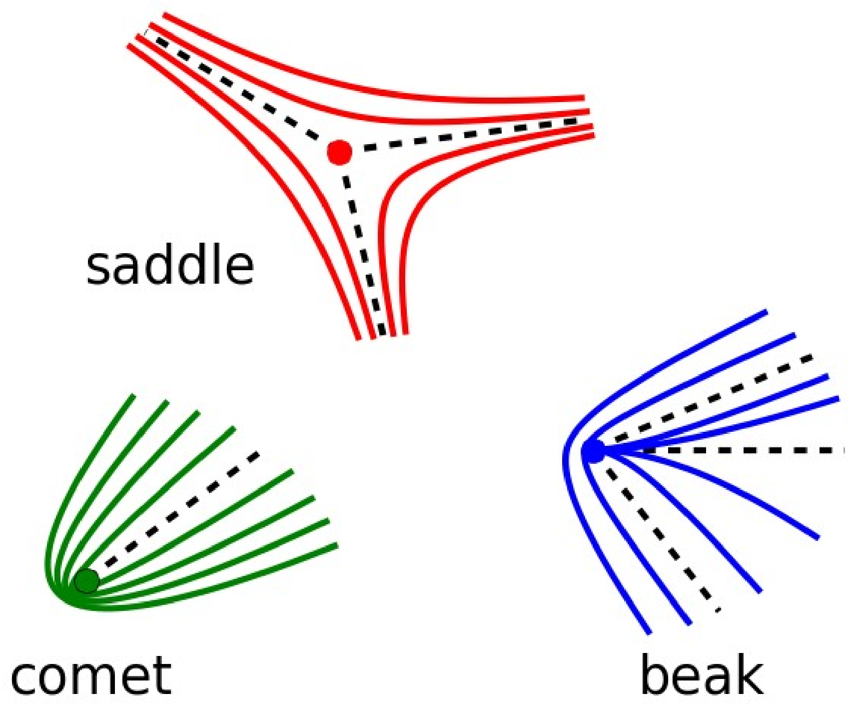
Unpolarized points:

$$p(x_0, y_0) = 0$$



3 types of unpolarized points:

$$p(x_0, y_0) = 0,\ \ \Delta x = x - x_0,\ \ \Delta y = y - y_0$$



3 types of unpolarized points:

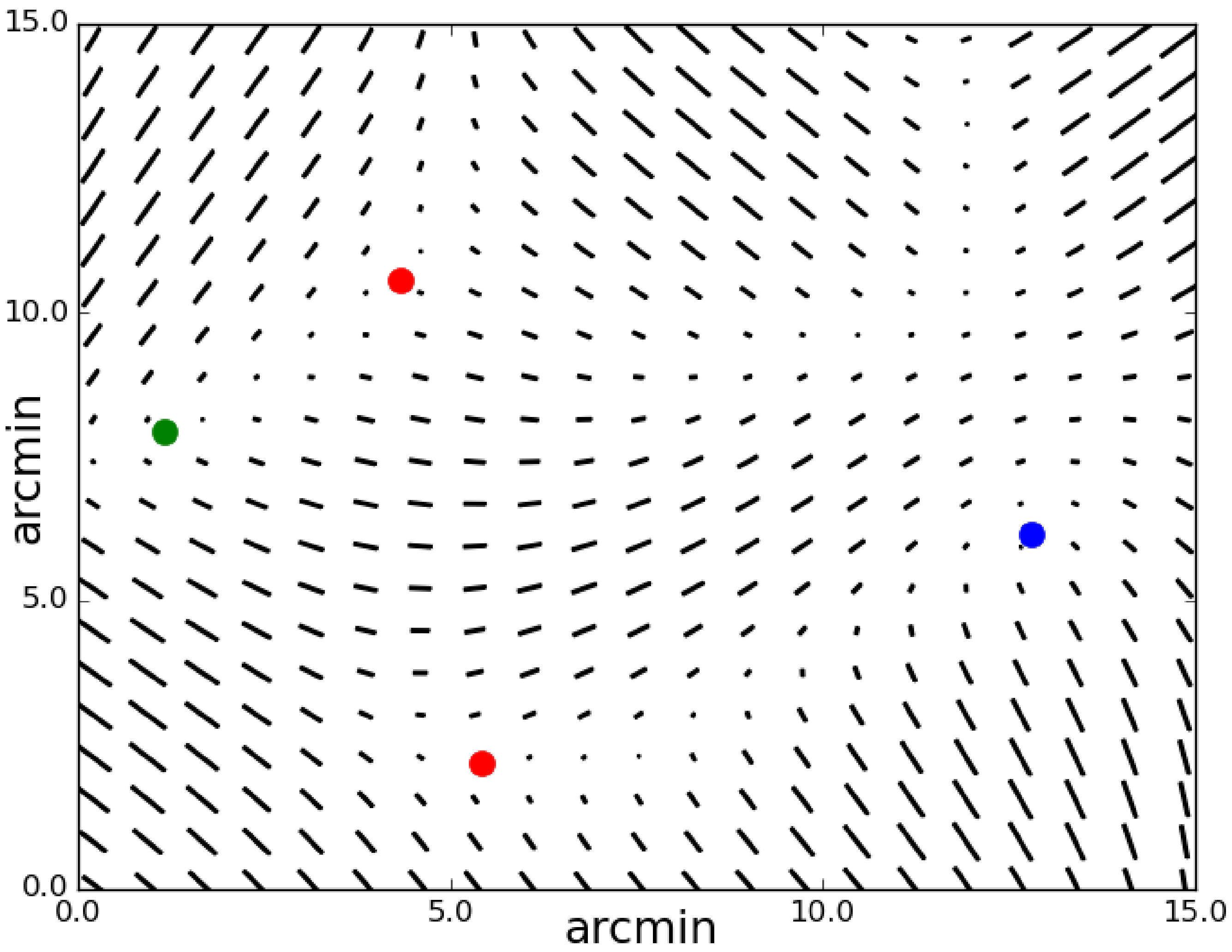
$$p(x_0, y_0) = 0, \ \ \Delta x = x - x_0, \ \ \Delta y = y - y_0$$

$$\begin{pmatrix} q \\ u \end{pmatrix} = \frac{\sigma_1}{\sigma_0} \cdot \begin{pmatrix} q_x & q_y \\ u_x & u_y \end{pmatrix} \cdot \begin{pmatrix} \Delta x \\ \Delta y \end{pmatrix}$$

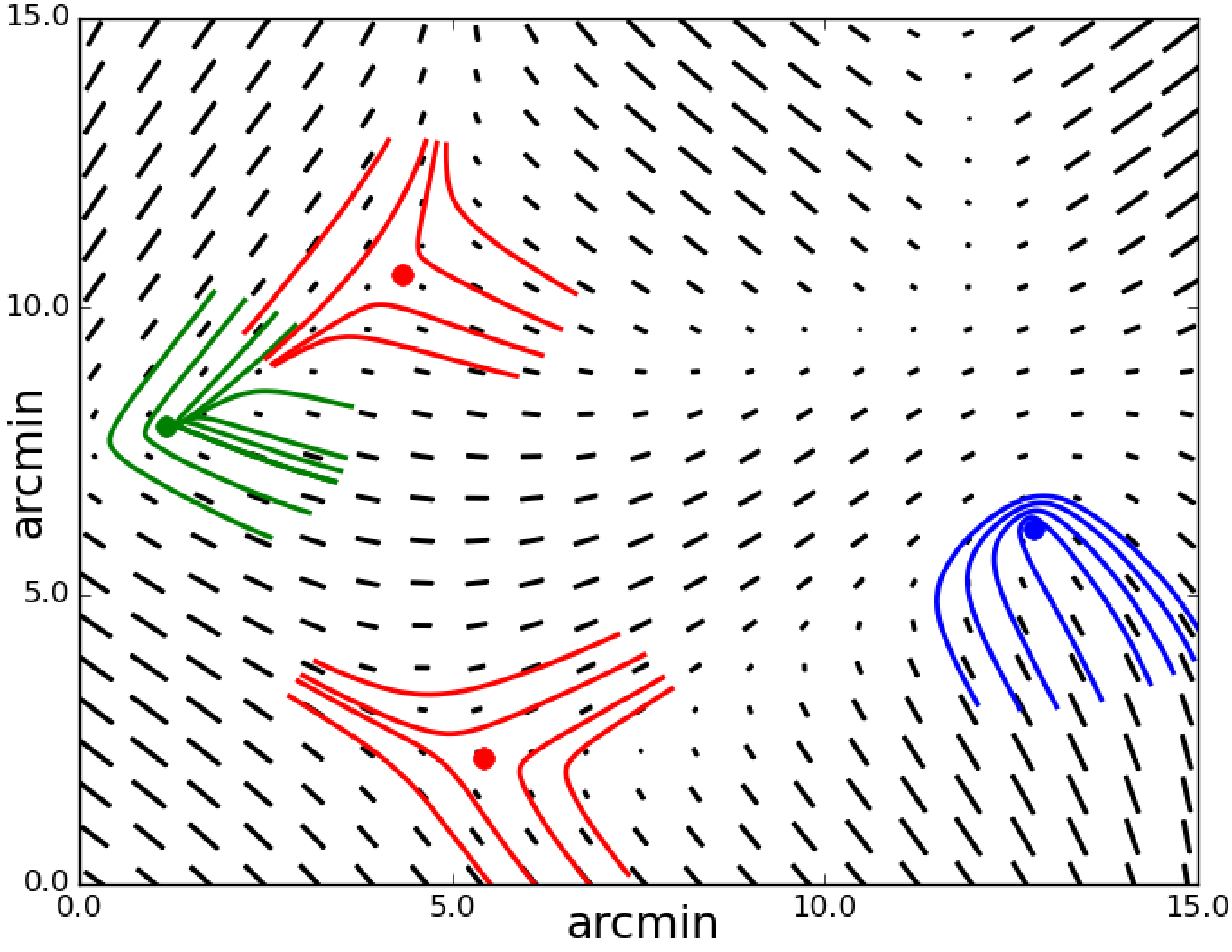
$$\begin{aligned} & d = q_x u_y - u_x q_y, \\ & D = 18abce - 4b^3 e + b^2 c^2 - 4ac^3 - 27a^2 e^2, \\ & a = u_y, \quad b = (u_x + 2q_y), \\ & c = (2q_x - u_y), \quad e = -u_x \end{aligned}$$

$$\begin{aligned} & d < 0, \quad \rightarrow \quad saddle, \\ & d > 0, D > 0 \quad \rightarrow \quad beak, \\ & d > 0, D < 0 \quad \rightarrow \quad comet \end{aligned}$$

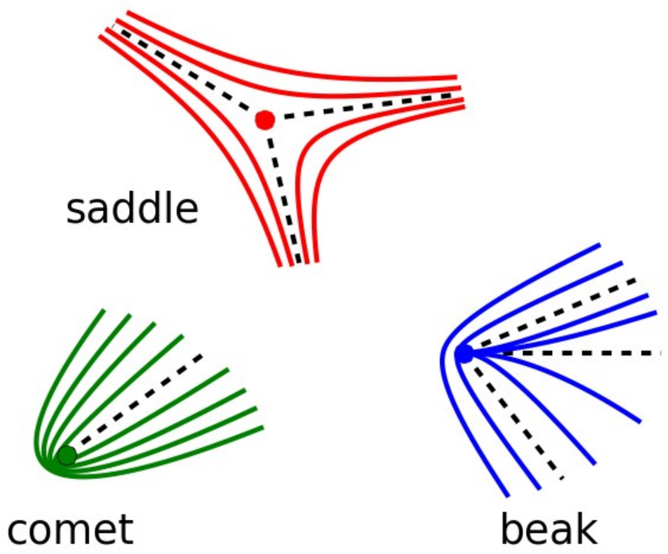
Unpolarized points

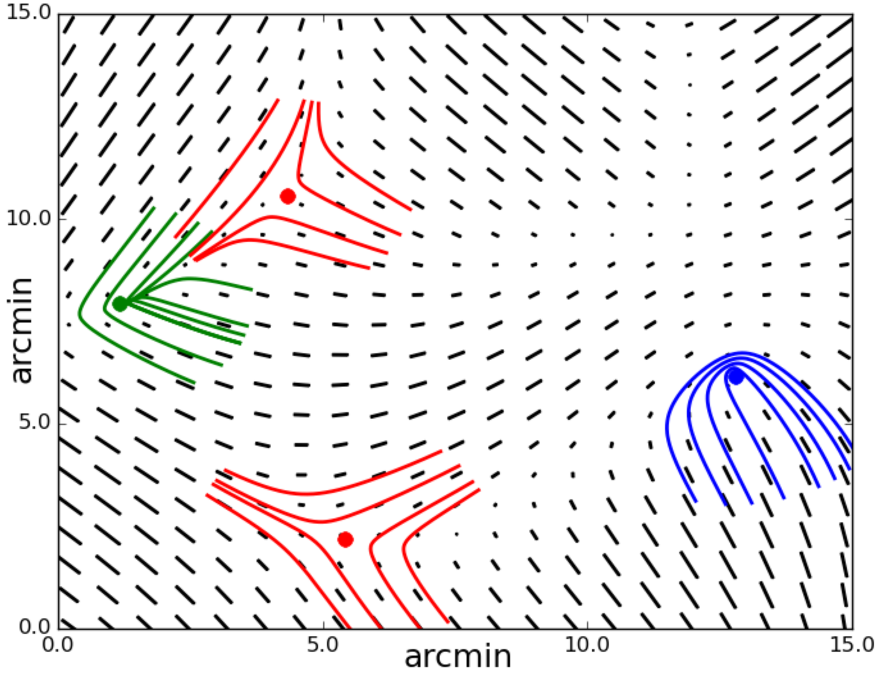


Unpolarized points



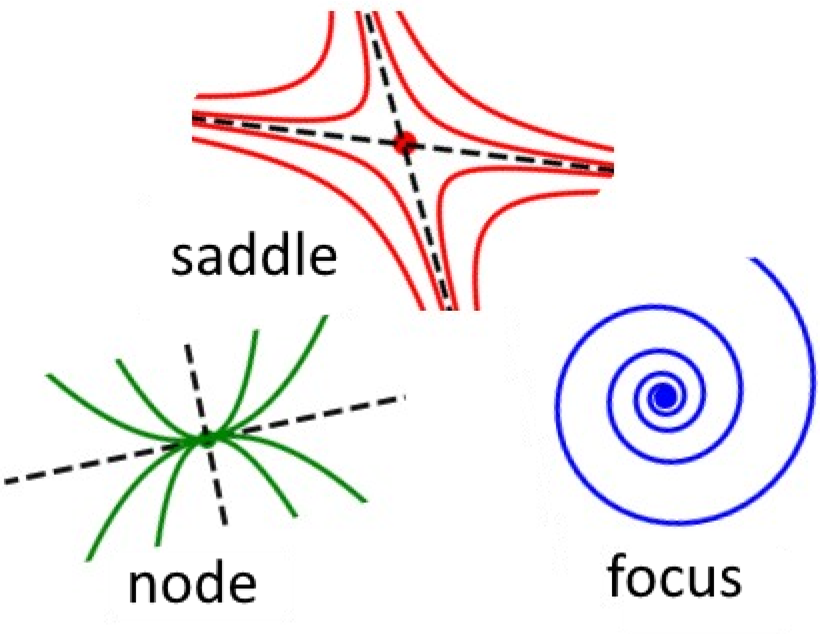
Unpolarized points



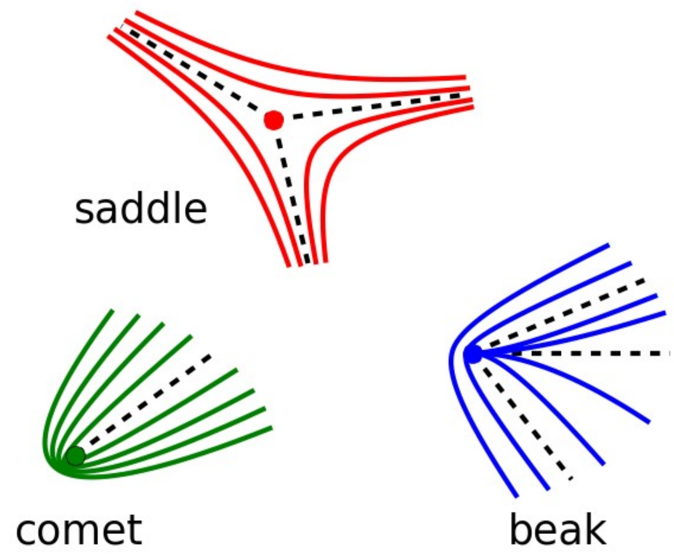


Zero points:

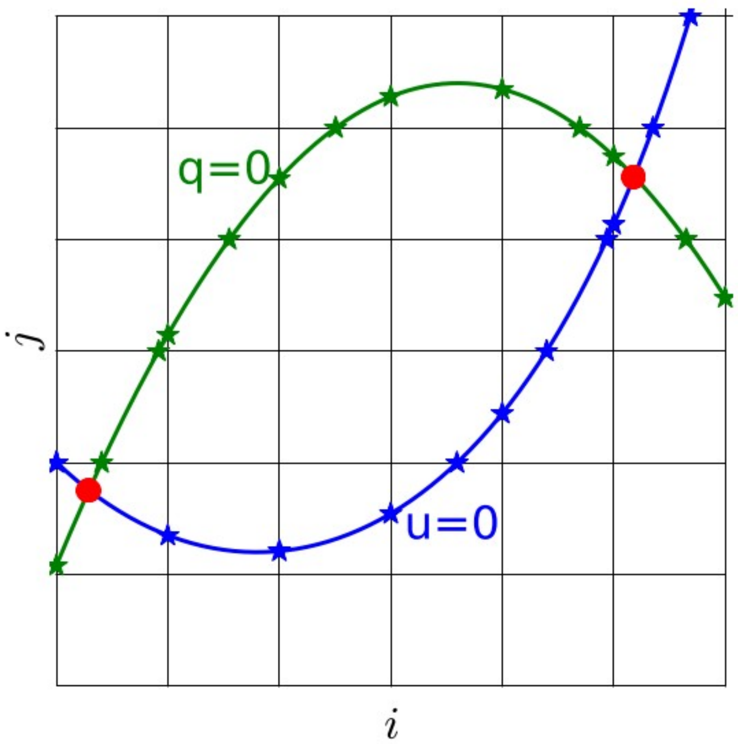
Vector field:

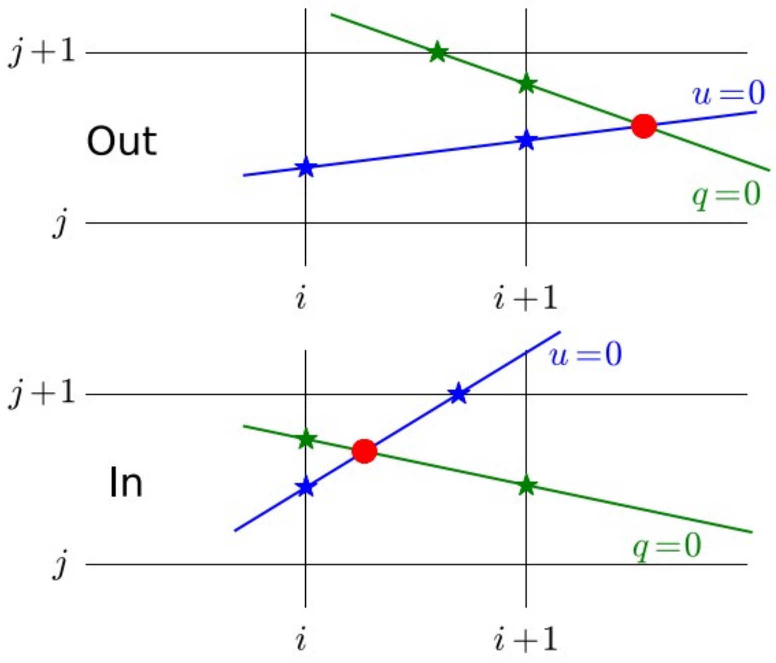


Tensor field (polarization):



Unpolarized points on a pixelized map





Joint probability function:

$$d\Phi = \frac{1}{2\pi^3} \cdot e^{-G} \cdot dq du dq_x dq_y du_x du_y,$$

$$G = \frac{q^2}{2} + \frac{u^2}{2} + q_x^2 + q_y^2 + u_x^2 + u_y^2.$$

$$\langle n \rangle = \frac{1}{2\pi^3} \frac{\sigma_1^2}{\sigma_0^2} \int e^{-g} \mid d \mid dq_x dq_y du_x du_y,$$

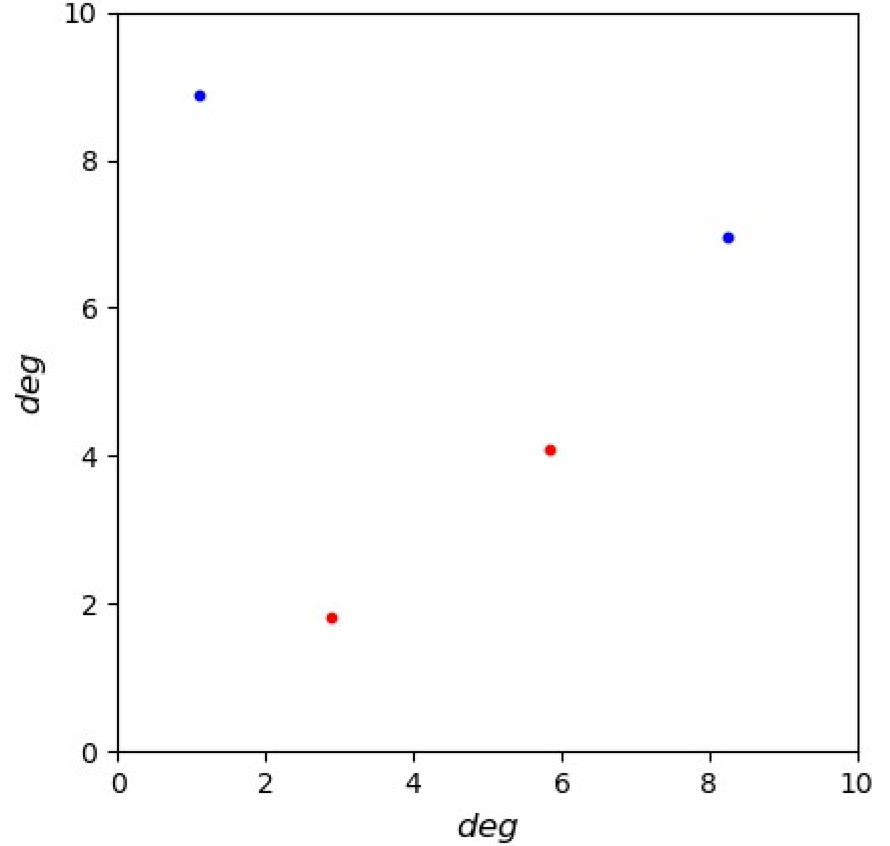
$$g = q_x^2 + q_y^2 + u_x^2 + u_y^2,$$

$$d = q_x u_y - u_x q_y.$$

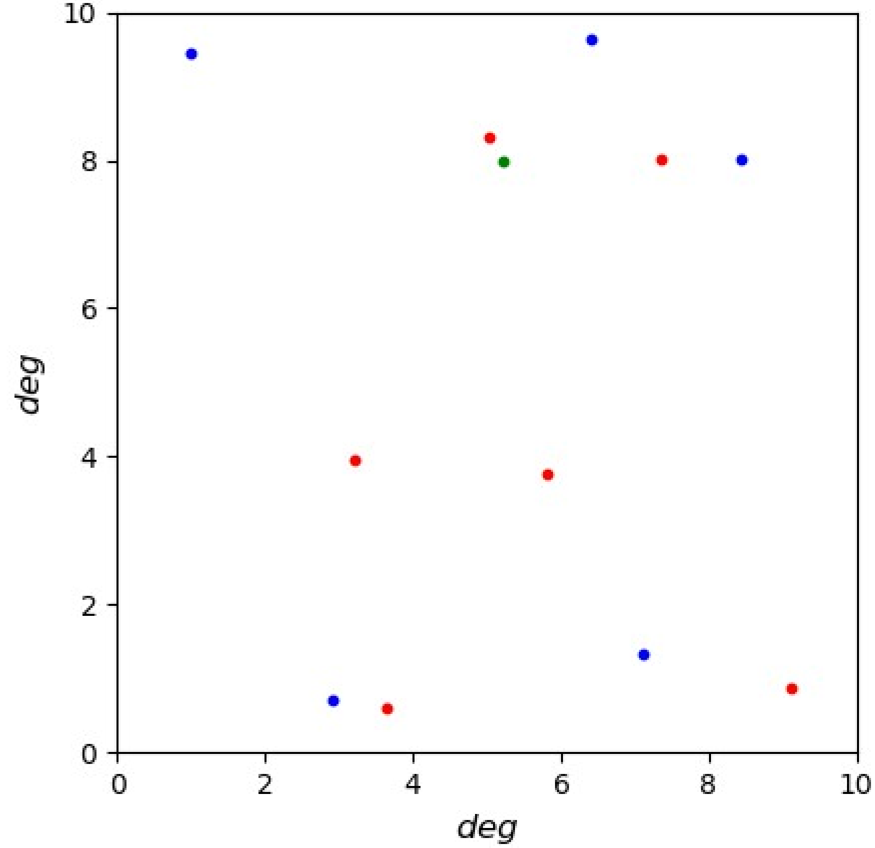
Distribution of unpolarized points:

In flat approximation:

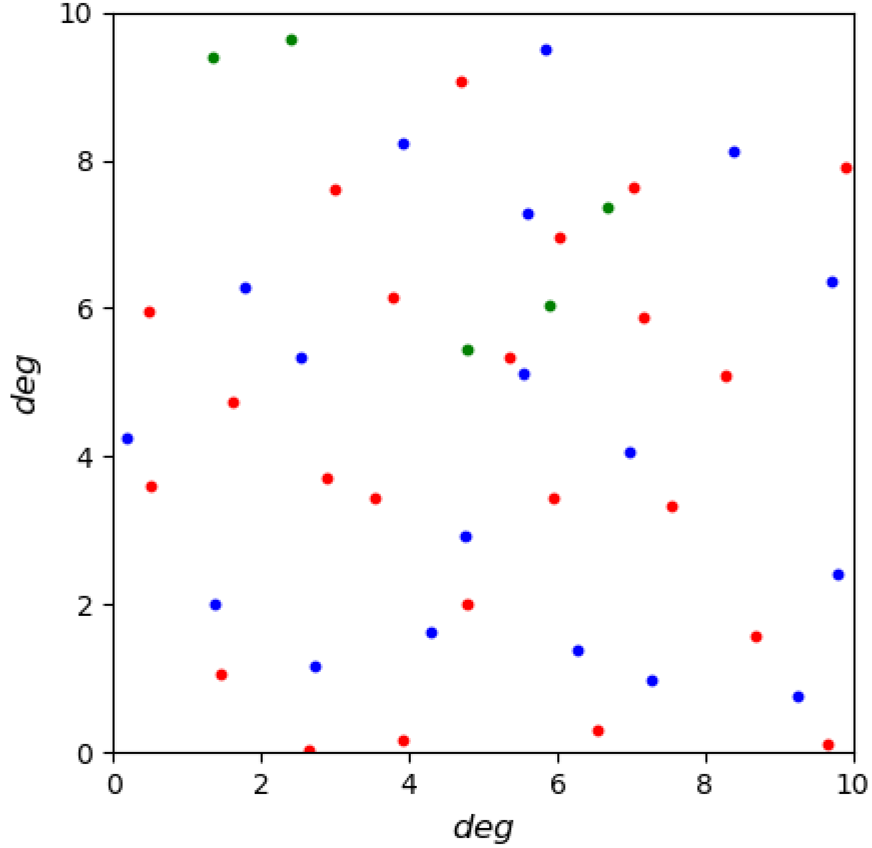
- $\langle n_{saddle} \rangle = 0.5 \langle n \rangle$
- $\langle n_{comet} \rangle \approx 0.447 \langle n \rangle$
- $\langle n_{beak} \rangle \approx 0.053 \langle n \rangle$



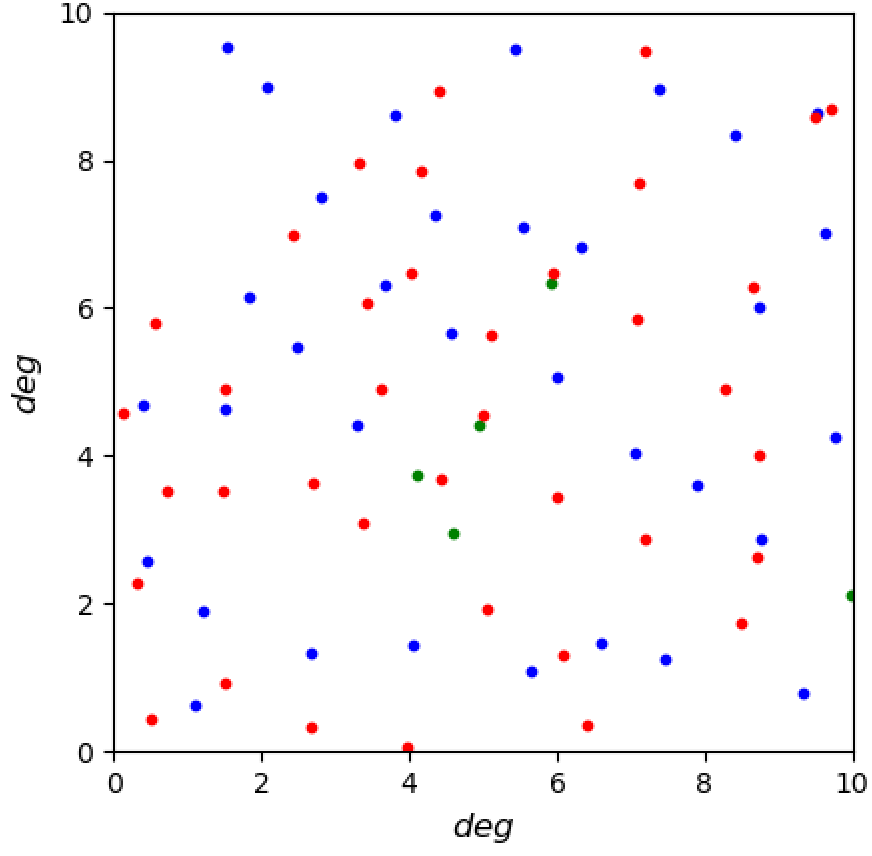
ℓ_{max}=100



ℓ_{max}=200



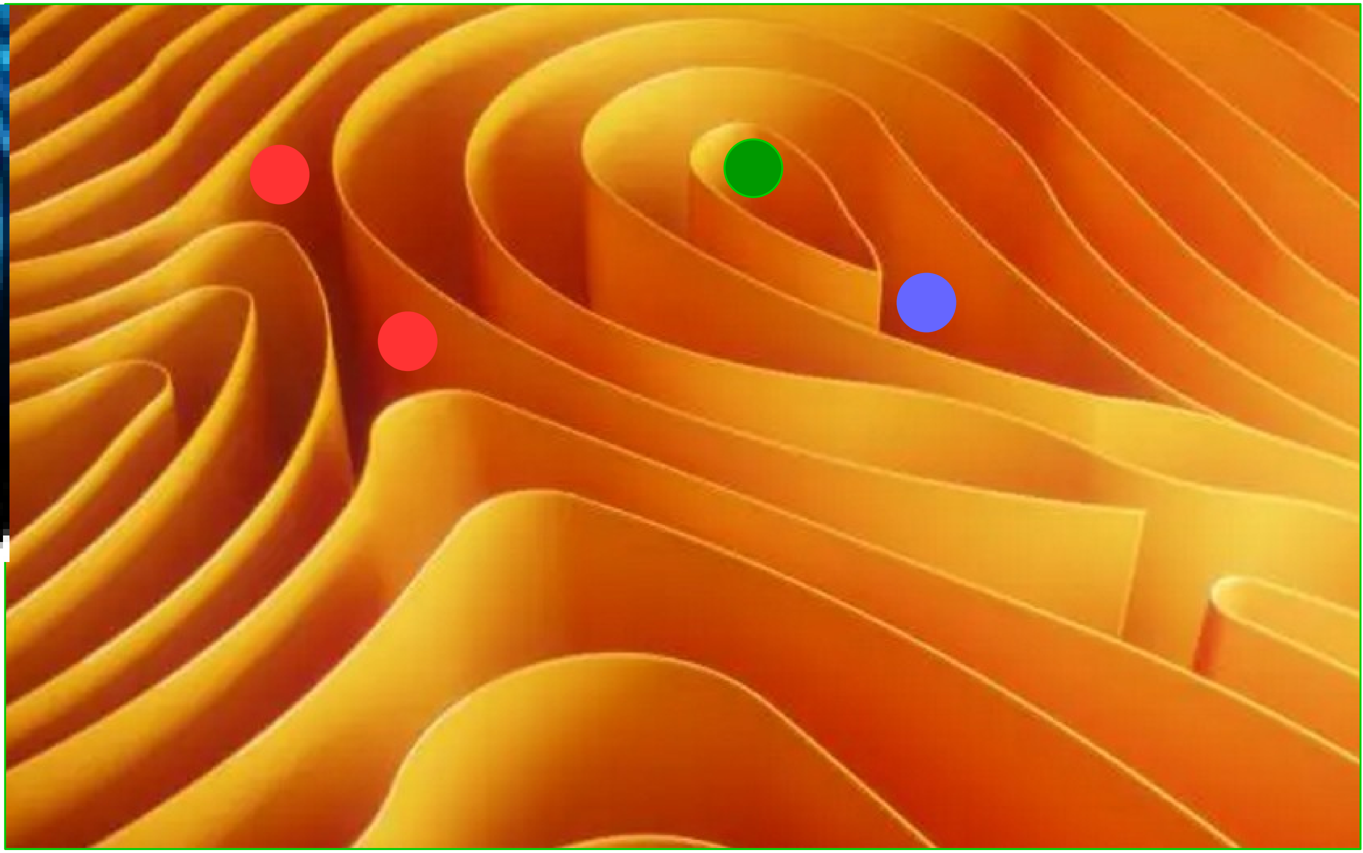
ℓ_{max}=300



ℓ_{max}=400

Fingerprints:

Fingerprints:



● (red)	saddle
● (green)	comet
● (blue)	beak

Planck polarization data analysis:

Let us start with simple example:

The Earth

Q and U from Earth heightmap:

$$E^{\oplus} = \sum_{\ell m} a_{\ell m}^{\oplus} \cdot Y_{\ell m}(\boldsymbol{\eta}), \quad \boldsymbol{\eta} = (\theta, \varphi),$$

$$Q^{\oplus} = \left[\frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2}\right] E^{\oplus}, \quad U^{\oplus} = 2\frac{\partial^2}{\partial x \partial y} E^{\oplus}.$$

$$Q^{\oplus} = \frac{1}{2} \sum_{\ell,m} \sqrt{\frac{(\ell+2)!}{(\ell-2)!}} \cdot a_{\ell m}^{\oplus} \left[{}_2Y_{\ell m}(\boldsymbol{\eta}) + {}_{-2}Y_{\ell m}(\boldsymbol{\eta})\right],$$

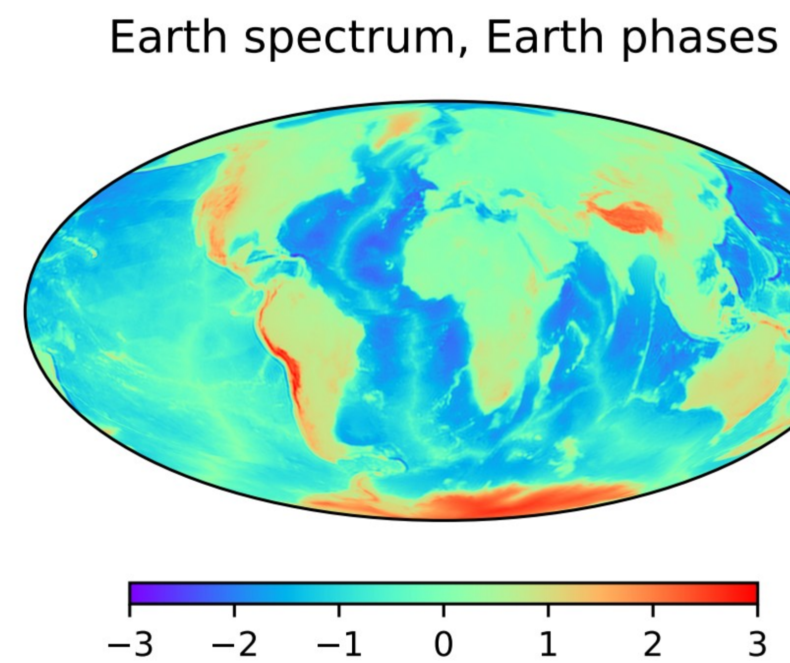
$$U^{\oplus} = \frac{1}{2i} \sum_{\ell.m} \sqrt{\frac{(\ell+2)!}{(\ell-2)!}} \cdot a_{\ell m}^{\oplus} \left[{}_2Y_{\ell m}(\boldsymbol{\eta}) - {}_{-2}Y_{\ell m}(\boldsymbol{\eta})\right].$$

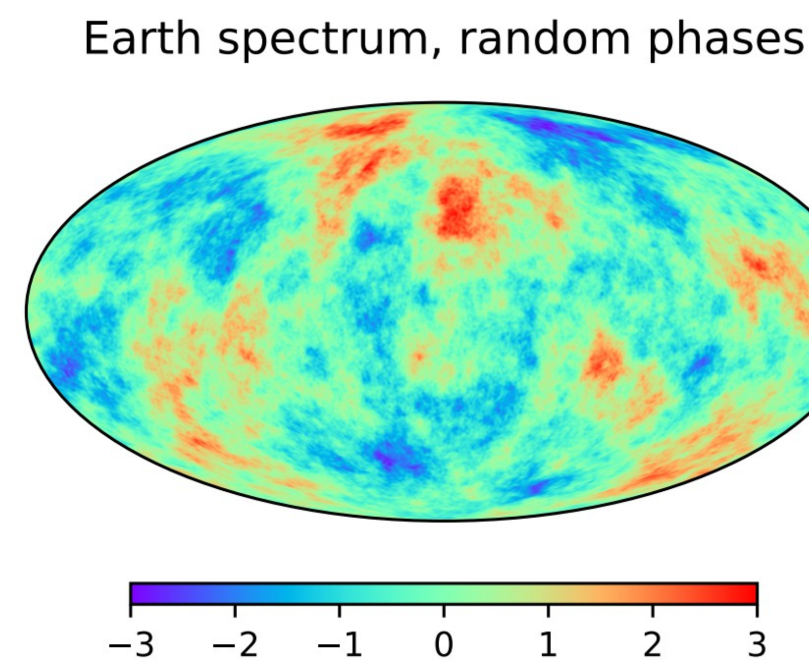
Gaussian map:

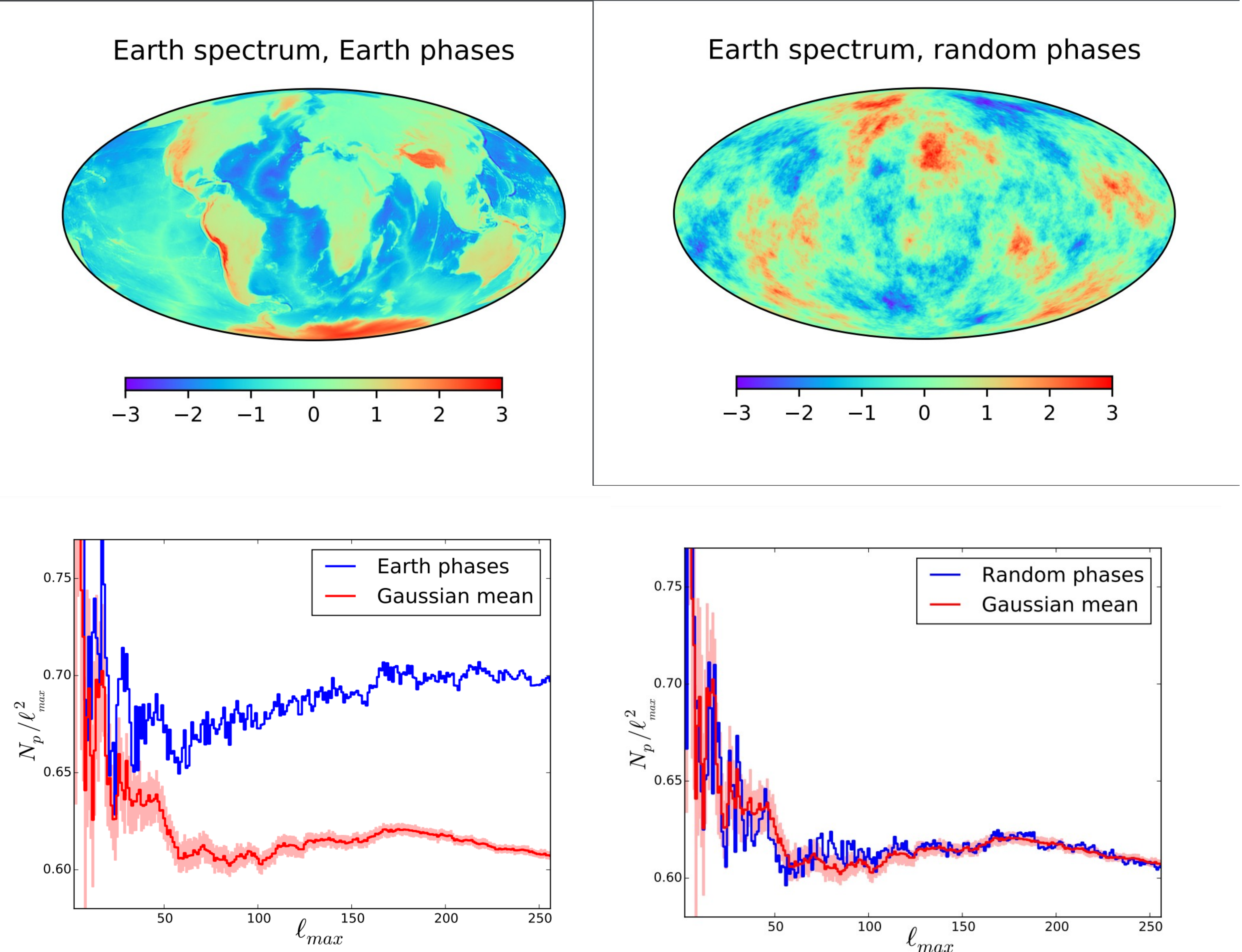
$$\widetilde{Q}^{\oplus} = \frac{1}{2} \sum_{\ell,m} \sqrt{\frac{(\ell+2)!}{(\ell-2)!}} \cdot \widetilde{a}_{\ell m}^{\oplus} \left[{}_2Y_{\ell m}(\boldsymbol{\eta}) + {}_{-2}Y_{\ell m}(\boldsymbol{\eta})\right],$$

$$\widetilde{U}^{\oplus} = \frac{1}{2i} \sum_{\ell,m} \sqrt{\frac{(\ell+2)!}{(\ell-2)!}} \cdot \widetilde{a}_{\ell m}^{\oplus} \left[{}_2Y_{\ell m}(\boldsymbol{\eta}) - {}_{-2}Y_{\ell m}(\boldsymbol{\eta})\right],$$

$$\sum_{m=-\ell}^{\ell} \left(\widetilde{a}_{\ell m}^{\oplus}\right) \cdot \left(\widetilde{a}_{\ell m}^{\oplus}\right)^* = \sum_{m=-\ell}^{\ell} \left(a_{\ell m}^{\oplus}\right) \cdot \left(a_{\ell m}^{\oplus}\right)^*.$$

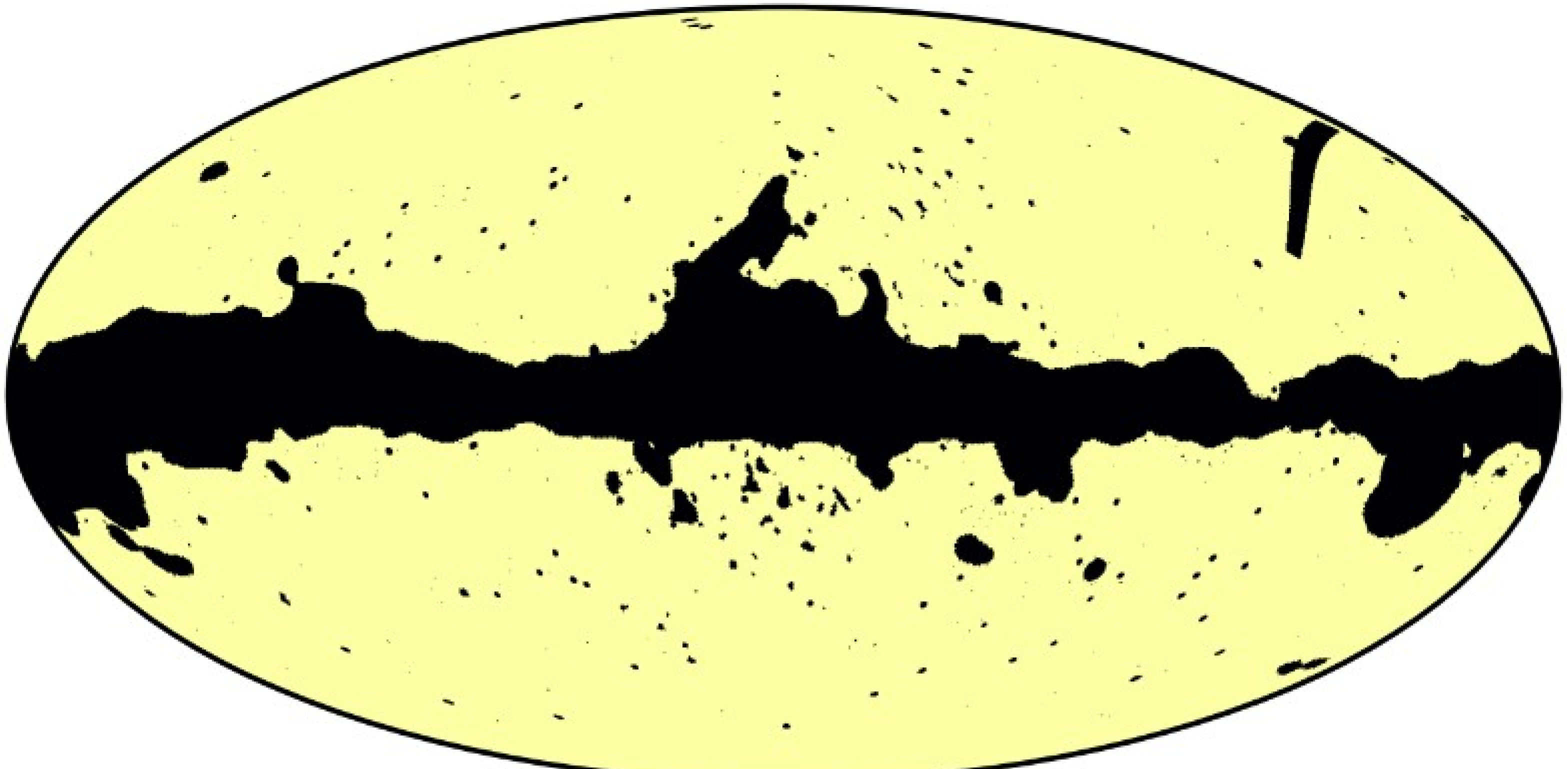

Earth spectrum, Earth phases
−3
−2
−1
0
1
2
3

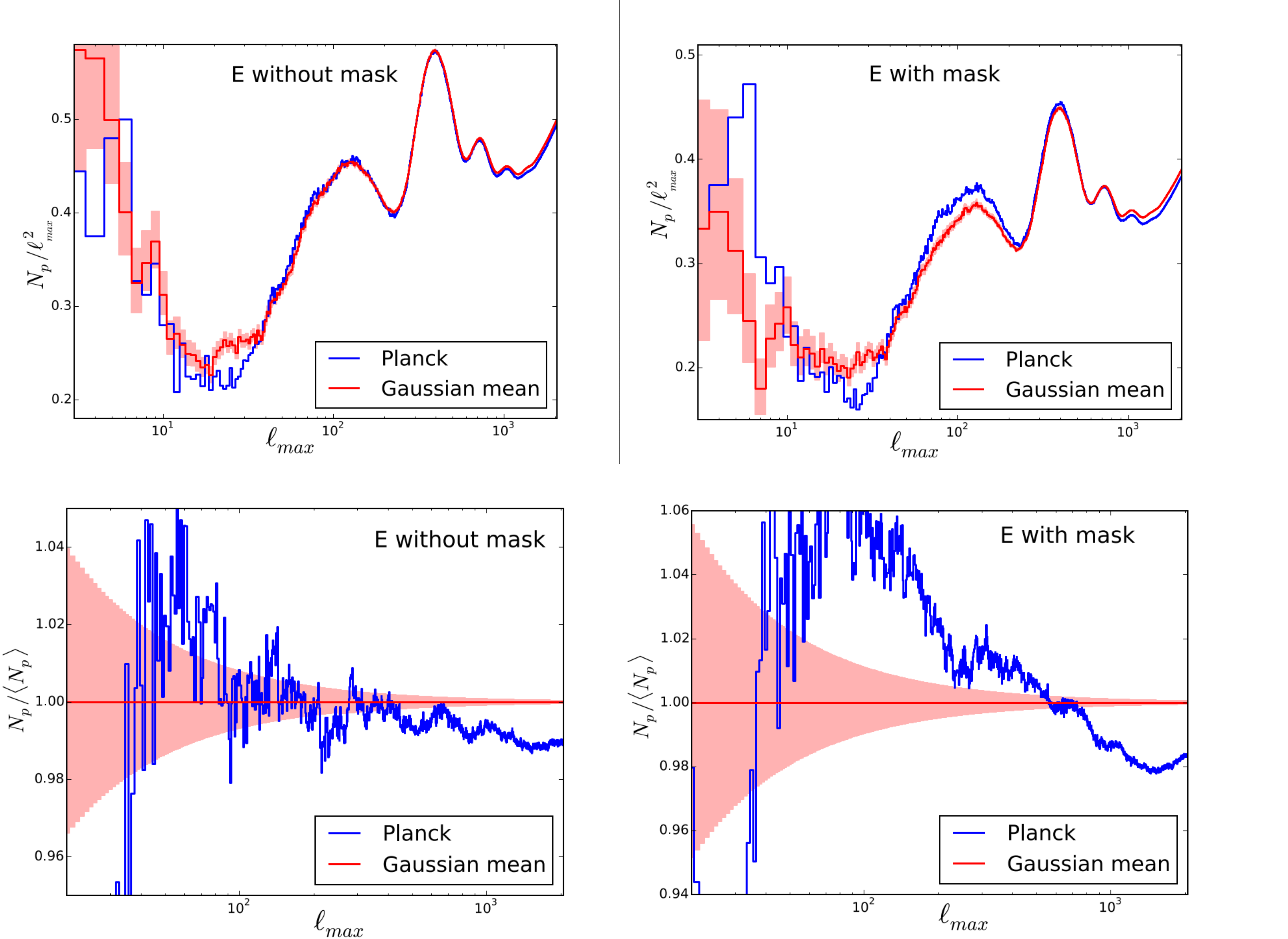

Earth spectrum, random phases
−3
−2
−1
0
1
2
3

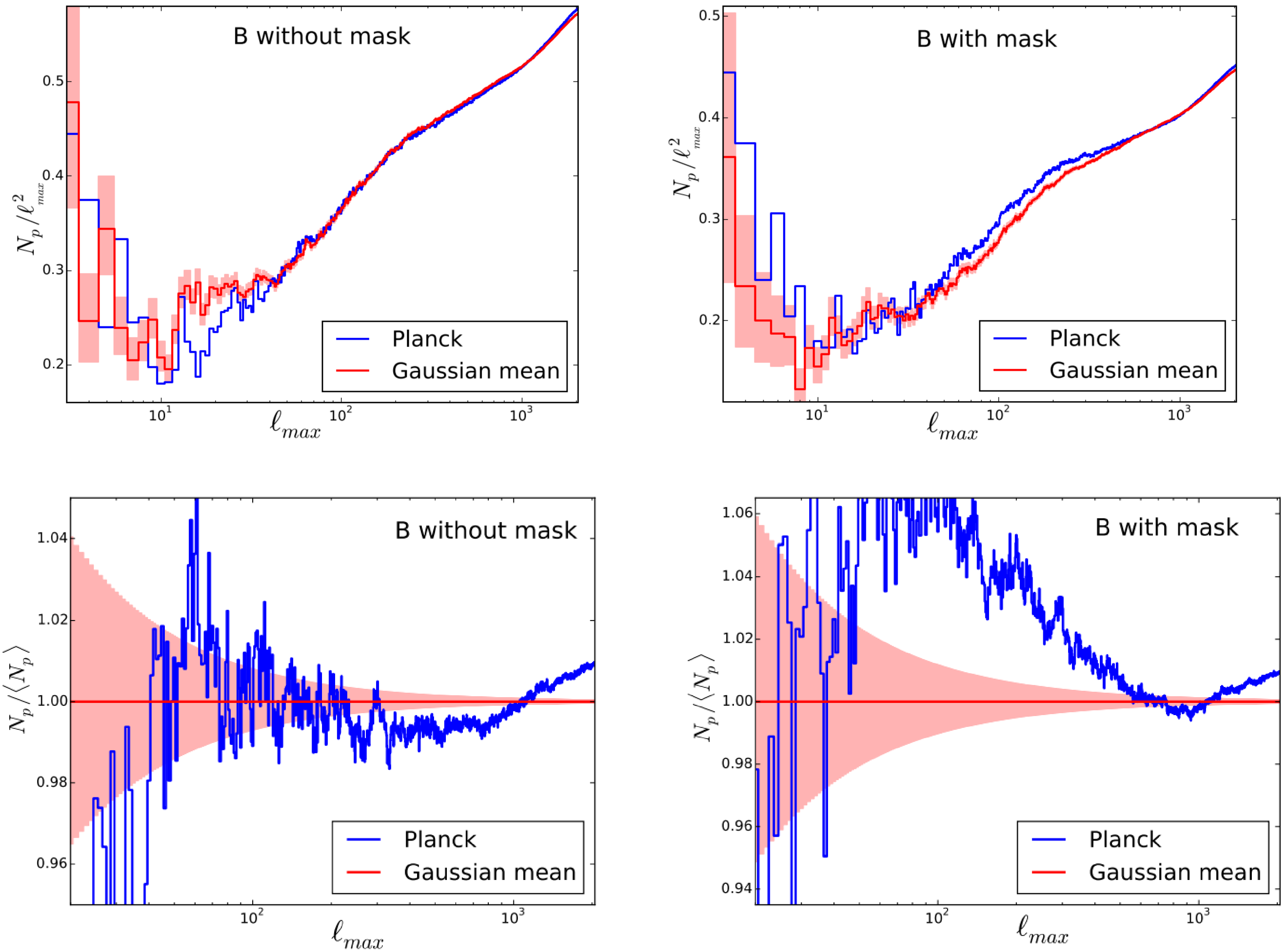


Earth spectrum, Earth phases
−3
−2
−1
0
1
2
3
Earth spectrum, random phases
−3
−2
−1
0
1
2
3
Earth phases
Gaussian mean
0.75
0.70
0.65
0.60
50
100
150
200
250
N_p/ℓ^2_{max}
ℓ_{max}
Random phases
Gaussian mean
0.75
0.70
0.65
0.60
50
100
150
200
250
N_p/ℓ^2_{max}
ℓ_{max}

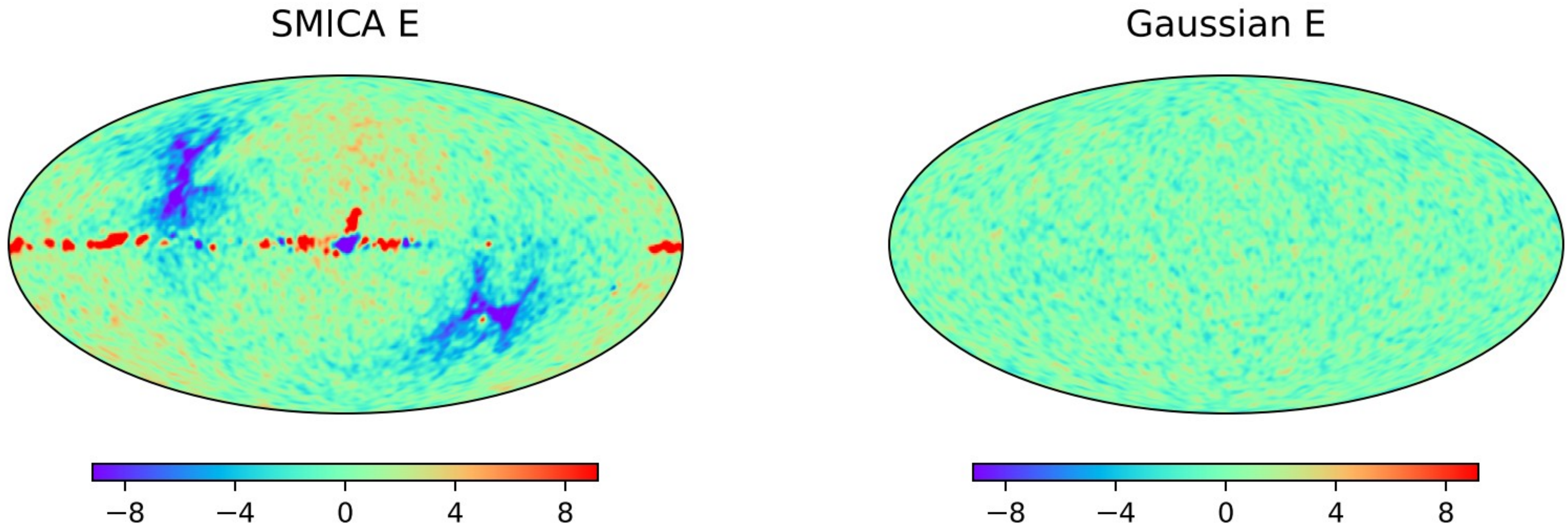
Polarization confidence mask




E without mask
Planck
Gaussian mean
N_p/ℓ_{max}^2
ℓ_{max}
0.2
0.3
0.4
0.5
10^1
10^2
10^3
E with mask
Planck
Gaussian mean
N_p/ℓ_{max}^2
ℓ_{max}
0.2
0.3
0.4
0.5
10^1
10^2
10^3
E without mask
Planck
Gaussian mean
$N_p/\langle N_p \rangle$
ℓ_{max}
0.96
0.98
1.00
1.02
1.04
10^2
10^3
E with mask
Planck
Gaussian mean
$N_p/\langle N_p \rangle$
ℓ_{max}
0.94
0.96
0.98
1.00
1.02
1.04
1.06
10^2
10^3

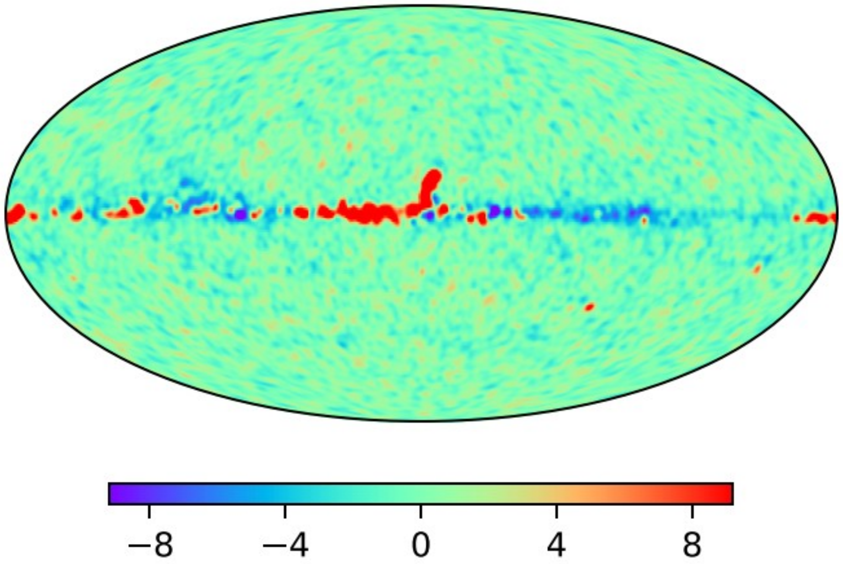

B without mask
Planck
Gaussian mean
N_p/ℓ_{max}^2
ℓ_{max}
B with mask
Planck
Gaussian mean
N_p/ℓ_{max}^2
ℓ_{max}
B without mask
Planck
Gaussian mean
$N_p/\langle N_p \rangle$
ℓ_{max}
B with mask
Planck
Gaussian mean
$N_p/\langle N_p \rangle$
ℓ_{max}

Planck E-polarization data analysis

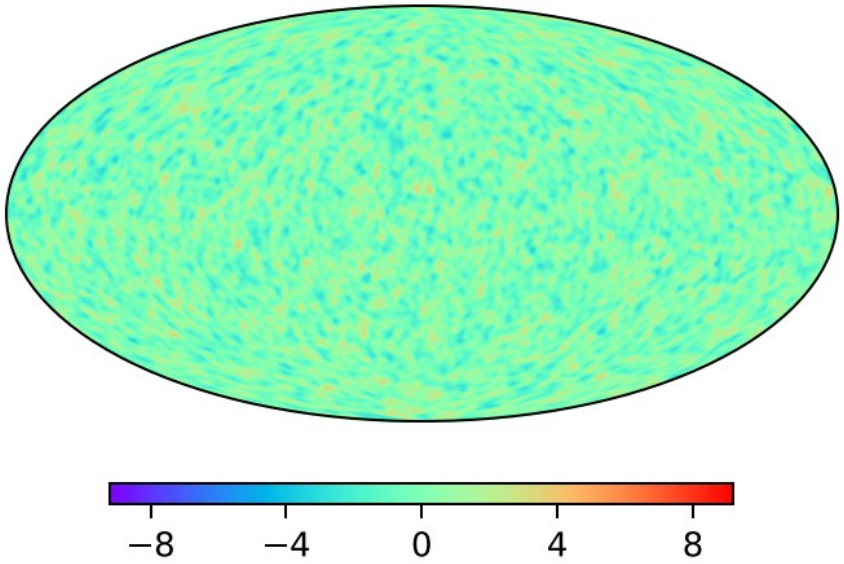


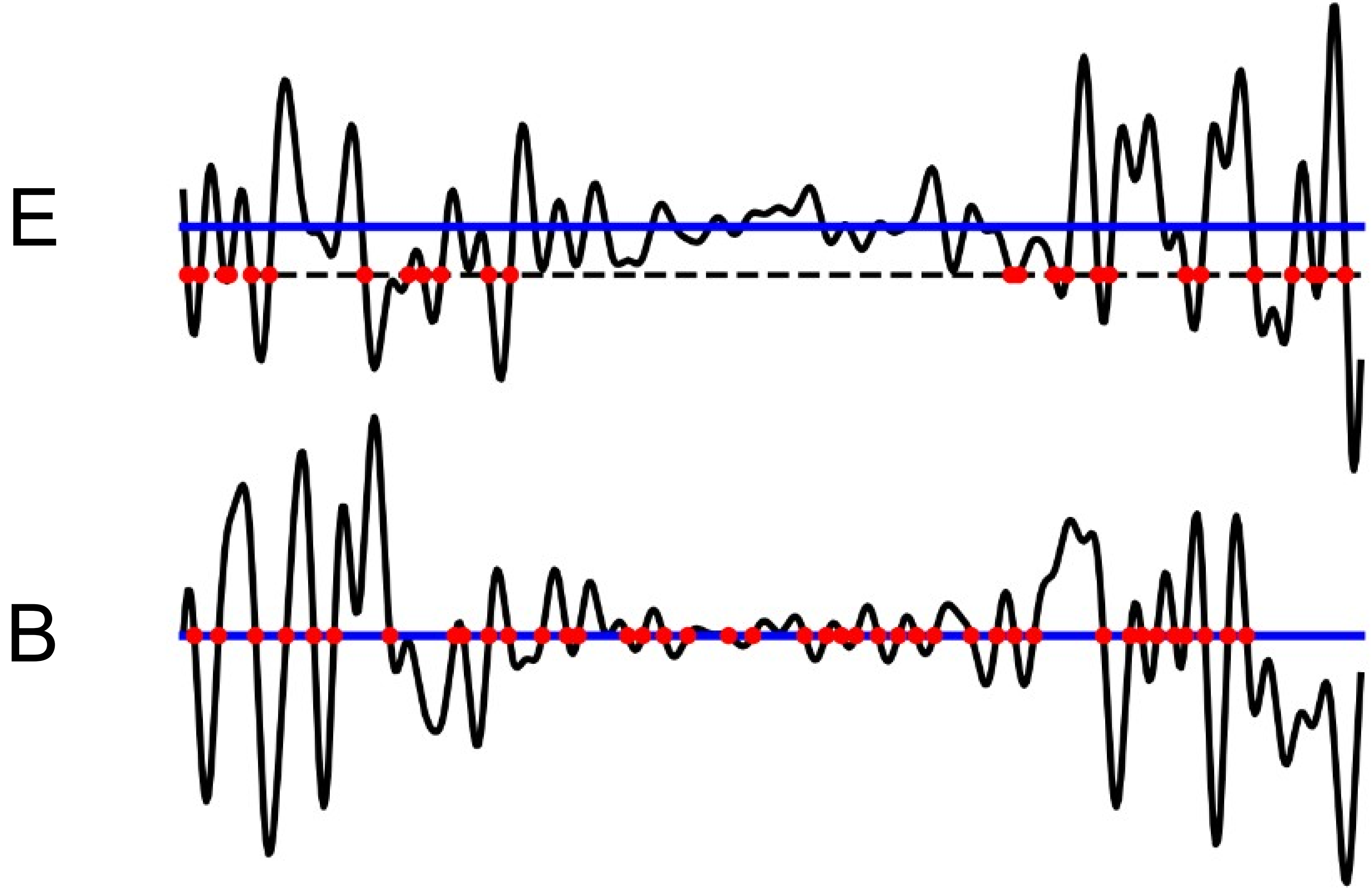
Planck B-polarization data analysis

SMICA B



Gaussian B




E
B

Conclusions

- The approach for studying the statistics of non-polarized points on the sky that we presented allows us to test for Gaussianity of the polarization maps at different angular scales and detect local non-Gaussianities
- The obvious non-Gaussianity of the E component of polarization that we discovered means that more correct data processing is needed to clear the polarization map of foregrounds.
- Taking into account the absence of experiments with good sensitivity to date, we need more advanced methods of filtering maps and careful ways of working with unevenly distributed white noise.
- It seems extremely interesting to study the statistics of unpolarized points of different types (saddles, comets and beaks)