Динамические кротовые норы во Вселенной и возможность их наблюдения

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Based on

- P.E. Kashargin, S.V. Sushkov, Collapsing Wormholes Sustained by Dustlike Matter, Universe **6**, 186 (2020)
- K.A. Bronnikov, P.E. Kashargin, S.V. Sushkov, Magnetized dusty black holes and wormholes, Universe **7**, 419 (2021)
- K. A. Bronnikov, P. E. Kashargin, S. V. Sushkov, Possible Wormholes in a Friedmann Universe, Universe **9**, 465 (2023)

Introduction

- A wormhole is a tunnel that connects two different regions of the same space-time, or two different space-times
- In the framework of GR, a static wormhole throat needs "exotic" matter that violates the Null Energy Condition (NEC)
- Wormholes have been considered in various extended (modified) theories of gravity and diverse models of (exotic) matter as a source of the geometry
- Besides spherically symmetric wormholes, those with diverse symmetries (cylindrical, axial, plane, rotating) have also been considered
- As well, dynamic wormholes have been considered in various aspects

- In this work we study the possible existence of traversable wormholes in GR with **classical** and **nonexotic** form of matter, widely used in various problems of astrophysics and cosmology, namely, dustlike matter, with or without an electromagnetic field.

Plan of the talk

- Overview of Tolman's solution
- Tolman's solution with a throat
- Particular wormhole model and their properties
- Geodesic properties and traversability of the dust wormhole
- The dust wormhole in a dust-filled Friedmann universe
- Estimations
- Summary

Tolman's solution with a magnetic field. I.

The spherically symmetric metric in a comoving reference frame for neutral dust particles:

$$ds^2 = d\tau^2 - e^{2\lambda(R,\tau)}dR^2 - r^2(R,\tau)d\Omega^2$$

The energy-momentum tensor (EMT) of dustlike matter:

$$T_\mu^{\nu[d]} = \rho u_\mu u^\nu$$

$(u^\nu) = (1, 0, 0, 0)$ is a velocity four-vector; ρ is the energy density

The EMT of the electromagnetic field:

$$T_\mu^{\nu[\mathrm{em}]} = \frac{q^2}{8\pi G r^4}\,\mathrm{diag}(1,\ 1,\ -1,\ -1),$$

q may be interpreted as an electric or magnetic charge in proper units

Tolman's solution with a magnetic field. II.

Einstein equations:

$$2r\ddot{r} + \dot{r}^2 + 1 - e^{-2\lambda}r'^2 = \frac{q^2}{r^2}, \quad (1)$$

$$\frac{1}{r^2}(1 + \dot{r}^2 + 2r\dot{r}\dot{\lambda}) - \frac{e^{-2\lambda}}{r^2}(2rr'' + r'^2 - 2rr'\lambda') = 8\pi G\rho + \frac{q^2}{r^4}, \quad (2)$$

$$\dot{r}' - \dot{\lambda}r' = 0. \quad (3)$$

Equation (3) is integrated in τ giving

$$\mathrm{e}^{2\lambda} = \frac{r'^2}{1 + f(R)}, \quad (4)$$

where $f(R)$ is an arbitrary function; $1 + f(R) > 0$.

Substituting (4) into (1) and integrating gives

$$\dot{r}^2 = f(R) + \frac{F(R)}{r} - \frac{q^2}{r^2}. \quad (5)$$

Tolman's solution with a magnetic field. III.

The physical meaning of $f(R)$:

$$\dot{r}^2 = f(R) + \frac{F(R)}{r} - \frac{q^2}{r^2}$$

$f(R) > 0$ – hyperbolic motion
$f(R) = 0$ – parabolic motion
$f(R) < 0$ – elliptic motion

The physical meaning of $F(R)$:
Substituting (4) and (5) into (2) yields

$$\rho = \frac{1}{8\pi G}\frac{F'(R)}{r^2 r'} \qquad \text{or} \qquad F(R) = 2GM(R) = 8\pi G \int \rho r^2 r' dR$$

Finally, depending on the sign of $f(R)$, the solution reads

$f > 0$ hyperbolic model :

$$\pm[\tau - \tau_0(R)] = \frac{1}{f}\sqrt{fr^2 + Fr - q^2} - \frac{F}{2f^{3/2}} \ln\left(F + 2fr + 2\sqrt{f}\sqrt{fr^2 + Fr - q^2}\right),$$

$f = 0$ parabolic model :

$$\pm[\tau - \tau_0(R)] = \frac{2\sqrt{Fr - q^2}(Fr + 2q^2)}{3F^2},$$

$f < 0$ elliptic model :

$$\pm[\tau - \tau_0(R)] = \frac{1}{h}\sqrt{-hr^2 + Fr - q^2} + \frac{F}{2h^{3/2}} \arcsin\frac{F - 2hr}{\sqrt{F^2 - 4hq^2}}, \quad h(R) := -f(R)$$

The elliptic model ($h > 0$):

$$r = \frac{F}{2h}(1 - \Delta \cos\eta),$$

$$\pm[\tau - \tau_0] = \frac{F}{2h^{3/2}}(\eta - \Delta \sin\eta), \qquad \Delta = \sqrt{1 - \frac{4hq^2}{F^2}}, \ 0 < \Delta \leq 1.$$

The special case $\Delta = 1$ $(q = 0)$: The solution reduces to Friedmann's closed isotropic model filled with dust under the assumptions

$$F(\chi) = 2a_0 \sin^3\chi, \qquad h(\chi) = \sin^2\chi, \qquad a_0 = \mathrm{const}$$

(the radial coordinate $R = \chi$ is here a "radial angle" of a 3D sphere), and we have

$$r = r(\eta, \chi) = a(\eta)\sin\chi, \qquad a(\eta) = a_0(1 - \cos\eta),$$

$a(\eta)$ being the cosmological scale factor.

Wormhole solution: Throat conditions. I.

Notice: $\rho \sim F'/r' > 0 \quad \Rightarrow \quad r' = 0$ is possible!

The existence of regular minimum values of r (at given τ) can be interpreted as throats.

Definition:

A **throat** is a regular minimum of the spherical radius $r(R,\tau)$ at given τ (that is, in a fixed spatial section of our space-time).

Throat conditions at $\tau = const$:

$$dl^2_{(3)} = \frac{r'^2 dR^2}{1+f(R)} + r^2(R)d\Omega^2 = dl^2 + r^2(l)d\Omega^2,$$

where $r(R) = r(R,\tau)|_{\tau=\mathrm{const}}$ and $dl = r'dR/\sqrt{1+f(R)}$

$$\mathrm{I.}\ \frac{dr}{dl} = 0, \qquad \mathrm{II.}\ \frac{d^2r}{dl^2} > 0$$

Wormhole solution: Throat conditions. II.

I. $\dfrac{dr}{dl} = \sqrt{1 + f(R_{\rm th})} = 0 \quad \Rightarrow \quad f(R_{\rm th}) = -1 \quad \text{or} \quad h(R_{\rm th}) = 1$

- We obtain that $f < 0$ ($h > 0$), hence the only elliptic models can be suitable for describing wormholes.
- $1 + f = 1 - h > 0 \quad \Rightarrow \quad h(R_{\rm th})$ is the maximum; $h'_{\rm th} = 0,\ h''_{\rm th} < 0$.

II. $\left.\dfrac{d^2r}{dl^2}\right|_{R=R_{\rm th}} = -\left.\dfrac{h'}{2r'}\right|_{R=R_{\rm th}} > 0.$

The throat conditions **(I)** and **(II)** impose restrictions on the functions $F(R)$ and $h(R)$.

Wormhole solution: Apparent horizon

R- or T-regions ?!

$$r^{,\alpha} r_{,\alpha}\Big|_{R_{\rm th}} = \frac{1}{r^2}\sin^2\eta\left(\frac{F^2}{4} - q^2\right) \geq 0$$

- Generally, the wormhole's throat is located in a T-region!!!
- Hence a would-be asymptotically flat wormhole configuration that could be constructed from the Tolman solution cannot be traversable and, if matched with an external Reissner-Nordström or Schwarzschild solution at some value of R, actually represents a black hole.
- It is possible the existence of dynamic cosmological wormholes, which can be obtained if we mutch both sides of the wormhole with Friedmann-Robertson-Walker asymptotics.

Particular wormhole model. I.

Specific choice for $h(R)$ and $F(R)$:

$$h(R) = \frac{1}{1+R^2}, \qquad F(R) = 2b(1+R^2)^k, \qquad b = \text{const} > 0$$

Wormhole solution with the throat located at $R = 0$:

$$r(R,\eta) = b(1+R^2)^2(1-\Delta\cos\eta), \qquad r'(R,\eta) = \frac{bR(1+R^2)(2N_2-N_1)}{\Delta(1-\Delta\cos\eta)}$$

$\Delta = \sqrt{1-q^2/b^2(1+R^2)^3}$
$N_1(R,\eta) = \cos\eta - 3\Delta + 3\Delta^2(\eta\sin\eta+\cos\eta) + \Delta^3(-2+\cos^2\eta)$
$N_2(R,\eta) = -\cos\eta + 2\Delta - \Delta^2(\cos\eta + \eta\sin\eta)$

$$\rho(R,\eta) = \frac{\Delta}{2\pi G b^2(1+R^2)^5(1-\Delta\cos\eta)(2N_2-N_1)},$$

$$\left.\frac{d^2r}{dl^2}\right|_{R=0} = \left.\frac{\Delta\,(1-\Delta\,\cos\eta)}{b(2N_2-N_1)}\right|_{R=0}.$$

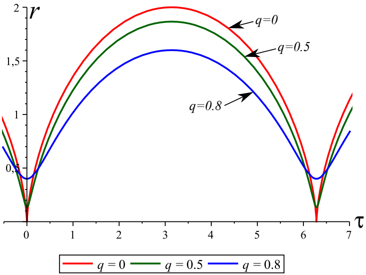


Рис.: Time dependence of the throat radius for $q = 0, 0.5, 0.8$ and in terms of τ.

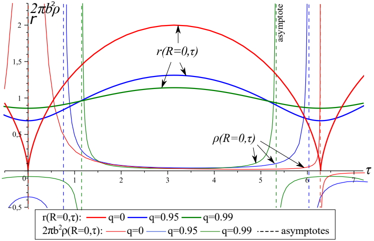
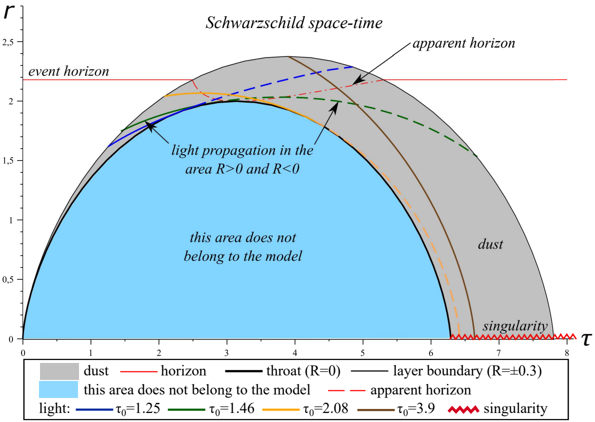


Рис.: Time dependence of the functions $2\pi b^2\rho$ (thin lines) and $r(0,\tau)$ (thick lines) on the throat $R = 0$ for $q = 0,\ 0.95,\ 0.99$. Dashed lines show the asymptotes. For other values of q the plots look in a similar way.

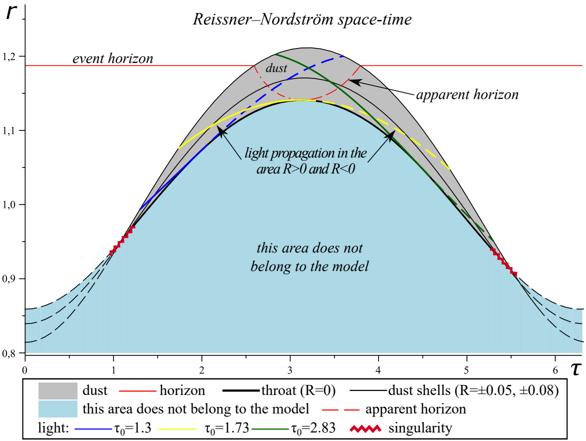
Traversability of the dust wormhole

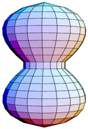
The equation of radial motion of a photon in the dust layer:

$$\frac{dr(\tau,R(\tau))}{d\tau}=\frac{1}{\sqrt{1+R^2}}\left(\frac{\Delta\sin\eta}{1-\Delta\cos\eta}-R\right)$$



Traversability of the dust wormhole





A "dumbbell wormhole": Formed by two closed Friedmann universes connected by a narrow neck

Friedmann closed model: $r(\chi,\eta) = a_0 \sin\chi(1-\cos\eta)$

Tolmen wormhole solution: $r(R,\eta) = b(1+R^2)^2(1-\cos\eta), \quad q=0$

The junction surface: $\chi = \chi^*$ and $R = R^*$

The junction conditions: $R^* = \pm\cot\chi^*, \qquad b = a_0 \sin^5\chi^*$

Notice: The whole composite model thus consists of two evolving closed Friedmann universes filled with dust and connected through a wormhole, forming a dumbbell-like configuration.

Some numerical estimations:

$a_0 \sim 10^{28}$ cm – the size of the visible part of the Universe
$r_{\rm th} \sim b \gg M_{Pl} \approx 10^{-33}$ cm; $r_{\rm th}$ – the throat radius

Then, using the junction conditions, we obtain

$$\sin\chi^* \gg 10^{-12}, \qquad R^* \ll 10^{12},$$

which implies $r^* = r(R^*) \gg 10^{16}$ cm ~ 0.01 light year.

- $r^* \sim 100$ light years (stellar cluster size) $\Rightarrow$ $\chi \sim 10^{-8}$ and $r_{\rm th} \sim 10^{20} l_{\rm pl} \sim 10^{-13}$ cm (a throat of atomic nucleus size)
- $r^* \sim 30\ {\rm kpc} \sim 10^{23}$ cm (galaxy size) $\Rightarrow$ $\chi \sim 10^{-5}$ and $r^* \sim 10^3$ cm (a throat of macroscopic size)

Estimations of density of the dust matter

The density:

$$\rho \sim \frac{10^{-30}}{(1+R^2)^5 \sin^{10}\chi^*} \ \frac{\mathrm{g}}{\mathrm{cm}^3}$$

- On the junction surface $R = R^*$, since $R^* \sim 1/\chi^*$, the density turns out to have a universal value independently from χ^*, $\rho \sim 10^{-30}\ \mathrm{g/cm^3}$ (of the order of mean cosmological density)
- On the throat $R = 0$:
  $\rho \sim 10^{50}\ \mathrm{g/cm^3}$ if $\chi^* = 10^{-8}$;
  $\rho \sim 10^{20}\ \mathrm{g/cm^3}$ if $\chi^* = 10^{-5}$

much more than the nuclear density!

Estimations. III.

Estimates of matter density $\rho_{\rm th}$ at the throat and the radius r_* of the wormhole region for different throat radii $r_{\rm th}$, in the case $k = 0.1$, $\rho_* = 1.2 \times 10^{-31}\ {\rm g/cm^3}$.

$r_{\rm th}$	r_*	$\rho_{\rm th}$ $[{\rm g/cm^3}]$
1.6×10^{-33} cm (L_{Pl})	1.6×10^4 km (Earth)	1.9×10^{91}
1 km	10^{21} cm $= 338$ pc	4.9×10^{15} (nuclear density)
10 km (neutron star)	700 pc	4.8×10^{13}
6.4×10^3 km (Earth)	5.1 Kpc	1.4×10^8 (white dwarf)
2.3×10^5 km	16 Kpc (Milky Way)	94×10^3
695×10^3 km (Sun)	23 Kpc	10^4
10^7 km (super BH)	52 Kpc	49
7×10^7 km	96 Kpc	1 (H_2O)
1 pc	5.7 Mpc	5.1×10^{-12}
6.5 pc	10 Mpc (galaxy cluster)	1.9×10^{-13}
10 Kpc	100 Mpc (void)	4.8×10^{-20} (interstellar medium)

Dynamics of the dust layers

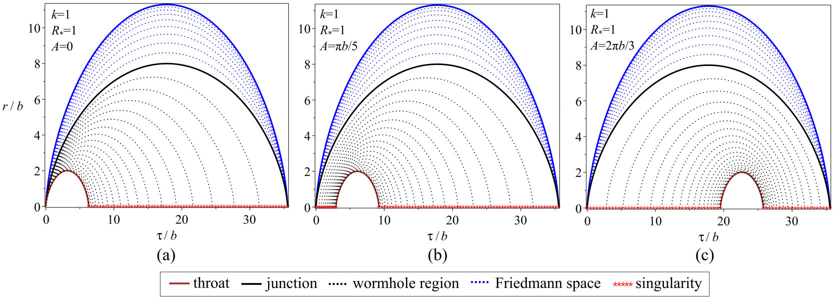
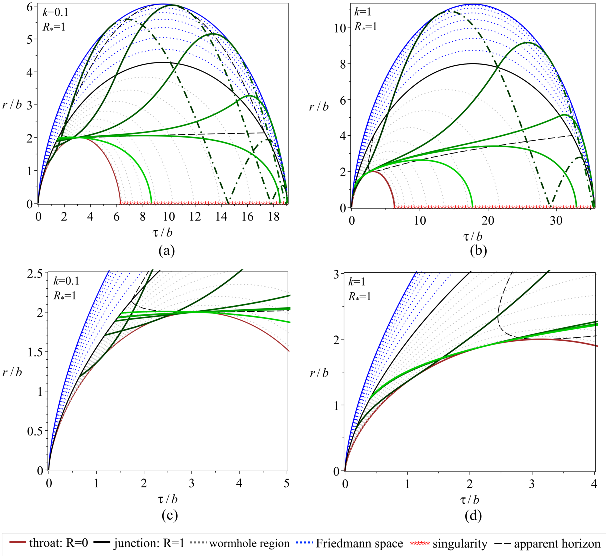
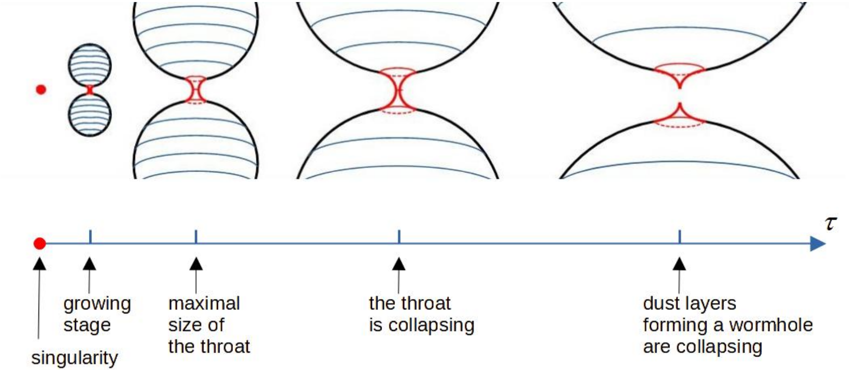


Рис.: The figure shows the dynamics of the dust layers in the case $k = 1$, $R_* = 1$ for different values of the parameters $A = 0$, $\pi b/5$, $2\pi b/3$.

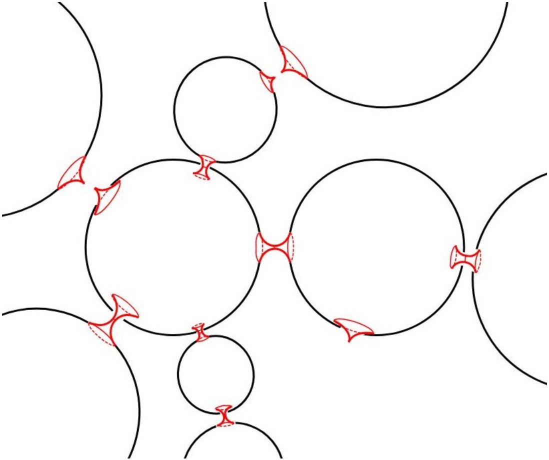
Travesability of dusty wormholes



An evolving wormhole connecting two Friedmann universes



Multiple wormholes in the multi-universe



Summary

- It has been shown that the q-Tolman dust clouds can contain wormhole throats under certain conditions on the arbitrary functions $f(R)$ and $F(R)$ of the general solution to the field equations.
- It has been shown that throats can only exist in the elliptic branch of q-Tolman space-times and are in general located in T-regions. Thus means that if a dust layer is matched to external Reissner-Nordström or Schwarzschild space-time regions, the whole configuration is a black hole rather than a wormhole.
- The q-Tolman space-times with throats are proven to exist for a finite period of time in their comoving reference frames.
- An analysis of radial null geodesics for particular examples of models under study has shown that the dust layers with throats can be traversable in both cases $q = 0$ and $q \neq 0$. In other words, a photon can cross such a dust layer more rapidly than this layer collapses.
- It has been shown that q-Tolman clouds with throats can form traversable wormholes in closed isotropic cosmological models filled with dust. These wormholes expand and contract together with the corresponding cosmological space-time.

THANKS FOR YOUR ATTENTION!